Resit Probability Theory (202001233) in M4-TCS/BIT (05-07-2024, 08.45-10.45 hr.)

Lecturers: A.K. Sinha & F. Kula **Module coordinator: Faiza Bukhsh** **A**

Last Name, First Name: ____________________

Student ID: ____________________

- This exam consists of 8 exercises.
- This document is your question as well as your answer paper.
- Write your answer in the designated boxes - try to keep your writing within the box.
- An extra box on the last page is given, in case you need extra space to finish a question. You must write the question number when you use the extra space of last page.
- Additional scratch paper is available for your convenience (not graded!)
- The formula sheet and the probability tables are provided separately. ***You must not write on them and should return them after the exam is over***.
- An ordinary calculator is allowed, not a programmable one (GR).
- Report numerical values to 3 decimal places where appropriate and unless mentioned otherwise.
- Please make your handwriting legible. Write only with blue or black ink. If you are using pencil, make sure that you use dark colour - light colour writing are very difficult to read. If we cannot read your writing, we shall not award points.

Do not use this page for computation or answer your question.
Only write your name and student ID in the designated space.

1	2	3	4	5	6	7	8	Total
5	3	2	2	4	8	10	6	40

Grade $= 1 + 9 \times \frac{\textbf{total score}}{40}$.

Part 1.

You do not have to show your works for the exercises of Part 1.

Exercise 1. [$5 \times 1 =$ **5 points.**]

In a university town, it is noticed that about 25% of the student-houses have a TV and about 45% have a refrigerator and 52% have a TV or a refrigerator or both. A student-house from the town is selected at random. Compute the following values (answer to **2 decimal places**):

a. P(the selected student-house has both TV and refrigerator) =

b. P(the selected student-house has neither a TV nor a refrigerator) =

c. P(the selected student-house has either a TV or a refrigerator but not both) =

d. What is the probability that the selected student-house has a TV given that it does not have a refrigerator?

e. Let T represents the event that "a student-house has a TV" and R represents the event that "a student-house has a refrigerator". In the current context, which of the following statement is true?

(**A**) T and R are not independent, because $P(T \cap R) \neq 0$.

(**B**) T and R are mutually exclusive, because $P(T \cap R) \neq P(T)\,P(R)$.

(**C**) T and R are not independent, because $P(T \cap R) \neq P(T)\,P(R)$.

(**D**) T and R are mutually exclusive, because $P(T \cap R) = 0$.

(**E**) None of the above.

Answer:

Exercise 2. [$3 \times 1 =$ **3 points**]

The probability that a person catches a cold during the cold and flu season is 0.4. Assume that 10 people are chosen at random. Compute following values (answer to **3 decimal places**):

a. What is the probability that four or more of them will catch a cold?

b. On average, how many of these ten people would you expect to catch a cold?

c. What is the standard deviation for the number of people catching a cold?

Exercise 3. [$2 \times 1 =$ **2 points**]

A person has recently joined a country club, whose monthly membership fee is €100. Additionally, he has to pay €30 per round of golf. The number of times he expects to play golf in a month is represented by a random variable with a mean of 10 and a standard deviation of 2.2.

a. What is his average monthly bill from the country club?

b. What is the standard deviation for his average monthly bill from the country club?

Exercise 4. [**2 points**. *All 4 parts correct: 2pt, 3 parts correct: 1pt, 2 parts correct: 0.5, otherwise 0 pt.*]

Suppose $X \sim N(0, 10)$ and $Y \sim N(0, 6)$. Moreover, X and Y are two independent random variables. Which of the following statements are true? Write **True** or **False** inside the box.

a. $(X - Y) \sim N(0, 4)$.

b. $P(X - Y \leq 0) = P(X \leq 0)\, P(Y \leq 0)$.

c. $P(X + Y \leq 0) = 0.6\, P(X \geq 0) + 0.4\, P(Y \geq 0)$.

d. $P(\min\{X, Y\} < 0) = 0.75$.

Exercise 5. [$1 + 1 + 2 =$ **4 points**]

Let X and Y be two random variables, where

$$E(X) = -9, \quad E(Y) = 2, \quad \mathrm{Var}(X) = 8, \quad \mathrm{Var}(Y) = 18, \quad E(XY) = -26.58.$$

Compute following values (answer to **3 decimal places**):

a. Covariance between X and Y, $\mathrm{Cov}(X, Y) =$

b. Correlation coefficient between X and Y, $\rho(X, Y) =$

c. $\mathrm{Cov}(X + Y, X - Y) =$

Part 2 starts from the next page.

Part 2

You must show your works for the exercises of Part 2. You must also motivate your answers where necessary.

Exercise 6. [**8 points**]

We have an unbiased die – two of its sides are labelled as 1, two other sides are labelled as 2 and the remaining two sides are labelled as 3. We roll the die twice and let the random variable X_i represent the result of the i^{th} roll $(i = 1, 2)$. Define $Y = X_1 - X_2$. Answer the following **showing necessary calculations**.

a. Determine $E(X_1)$ and $\mathrm{Var}(X_1)$. [3pt]

Exercise 6 contd.

b. Determine $P(Y = 0)$ and $P(|Y| = 1)$. [2pt]

c. Determine $E(Y)$ and $\text{Var}(Y)$. [3pt]

Exercise 7. [**10 points**]

Following is the probability density function of a random variable X:

$$f_X(x) = \begin{cases} 2(1-x), & \text{if } 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Answer the following (**show all relevant works and motivate where necessary**):

a. Determine c such that $P(X \leq c) = 0.64$. [2pt]

b. Compute $E(X)$ and $\mathrm{Var}(X)$. [3pt]

Exercise 7 contd.

c. Determine the range S_Y of $Y = \sqrt{X}$ and the density function $f_Y(y)$ on this range. [3pt]

d. Determine $E(Y)$, where $Y = \sqrt{X}$. [2pt]

Exercise 8. [**6 points**]

In order to assess traffic density on a road in Enschede the city council is measuring the time X (in minutes) between consecutive cars in southern direction. It is found that X is exponentially distributed with rate $\lambda = 0.48$.

a. Determine $P(X > 3)$. Answer to **3 decimal places**. [2pt]

b. A car passed at time $t = 0$. The council measures H, the time until 100 cars have passed (101 cars, including the car at $t = 0$). That means $H = X_1 + \ldots + X_{100}$, where X_i is the time between the i^{th} and the $(i+1)^{th}$ cars in southern direction $(i = 1, \ldots, 100)$. Give an approximate distribution of H and its parameters. State the necessary assumptions and also check all the conditions required to obtain the approximate distribution of H. [2pt]

Exercise 8 contd.

c. Determine $P(H > 350)$, where H is as defined in part **b.** Answer to **3 decimal places**. [2pt]

Extra writing space (can be used for any question as needed). Write the question number which you are answering here.